The automorphism tower problem for groups

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Automorphism Group

Begin with a group G.

An *automorphism* of G is an isomorphism of G with itself.

$$f : G \cong G$$

The collection of automorphisms of G forms a group, under composition. This is the *automorphism group* $\mathrm{Aut}(G)$.

We may thus iterate the process

$$G \quad \mathrm{Aut}(G) \quad \mathrm{Aut}(\mathrm{Aut}(G)) \quad \cdots$$

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The Automorphism Tower

Begin with a group G, and iteratively compute the automorphism group.

$$G \qquad \mathrm{Aut}(G) \qquad \mathrm{Aut}(\mathrm{Aut}(G)) \qquad \cdots$$

In fact, each group maps (homomorphically) into the next using inner automorphisms.

$$G \to \mathrm{Aut}(G) \to \mathrm{Aut}(\mathrm{Aut}(G)) \to \cdots$$

Specifically...

Inner automorphims

For any $g \in G$ we may define the *inner automorphism*

$$i_g(h) = ghg^{-1}$$

which is an automorphism of G.

We thereby obtain a natural map $\pi : G \to \mathrm{Aut}(G)$

$$\pi : g \mapsto i_g$$

- The kernel of π is precisely the center of G.
- The range of π is precisely $\mathrm{Inn}(G)$.

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We iteratively compute the automorphism group.

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a *direct limit*, a natural limit process.

Iterating Transfinitely

One can continue building the tower transfinitely, using the natural direct limit at limit stages.

$$G_0 \to G_1 \to G_2 \to \cdots \to G_\omega \to G_{\omega+1} \to \cdots$$

$$\cdots \to G_\alpha \to G_{\alpha+1} \to \cdots$$

Does it ever stop? The tower *terminates* if there is a fixed point, a group which is isomorphic to its automorphism group by the natural map. Such a group is complete: it is centerless and has only inner automorphisms.

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Automorphism Tower Problem

Question

Which groups have a terminating automorphism tower?

Question

For a given group, how long does it take for the tower to terminate?

An Important Special Case

If G is centerless, the tower simplifies considerably.

Since for any automorphism θ of G we have

$$\theta \circ i_g \circ \theta^{-1} = i_{\theta(g)},$$

it follows that $\mathrm{Inn}(G) \trianglelefteq \mathrm{Aut}(G)$ and, for G centerless, $C_{\mathrm{Aut}(G)}(\mathrm{Inn}(G)) = 1$.

In particular, if G is centerless, then $\mathrm{Aut}(G)$ is also centerless. Also $\mathrm{Aut}(\mathrm{Aut}(G))$ is centerless, and so on.

The Tower of a Centerless G

In fact, if the initial group G is centerless, then every group in the tower is centerless.

$$G \hookrightarrow G_1 \hookrightarrow \cdots \hookrightarrow G_\omega \hookrightarrow \cdots \hookrightarrow G_\alpha \hookrightarrow G_{\alpha+1} \hookrightarrow \cdots$$

Consequently, all the maps are injective. So we may identify each group with its image and view the tower as building up to larger and larger groups.

$$G \subseteq G_1 \subseteq \cdots \subseteq G_\omega \subseteq \cdots \subseteq G_\alpha \subseteq G_{\alpha+1} \subseteq \cdots$$

Outer automorphisms become inner automorphisms in the next group. Does the process ever close off?

The Classical Result

Classical Theorem (Wielandt, 1939)

The automorphism tower of any centerless finite group terminates in finitely many steps.

Question (Scott, 1964)

Is there a group whose automorphism tower never terminates?

Scott specifically mentions the possibility of transfinite iterations and towers of non-centerless groups.

Later Progress

Theorem (Rae and Roseblade, 1970)

The automorphism tower of any centerless Cernikov group terminates in finitely many steps.

Theorem (Hulse 1970)

The automorphism tower of any centerless polycyclic group terminates in a countable ordinal number of steps.

The Solution for Centerless G

Simon Thomas solved the Automorphism Tower Problem for centerless groups.

Theorem (Thomas, 1985, 1998)

The automorphism tower of any centerless group eventually terminates. Indeed, the automorphism tower of a centerless group G terminates in fewer than $(2^{|G|})^+$ many steps.

Fodor's Lemma lies at the heart of his elegant proof.

Answering the full question

Simon Thomas solved the automorphism tower problem for centerless groups: every centerless group has a terminating automorphism tower.

But what about the non-centerless groups?

In particular,

Question

Does every group have a terminating automorphism tower?

The answer is. . . . . .

Main Theorem

Main Theorem (Hamkins)

Every group has a terminating automorphism tower.

Proof: The strategy is to show that every automorphism tower leads eventually to a *centerless* group,

$$G \to G_1 \to \cdots \to G_\alpha \to \cdots \to G_\beta \to \cdots$$

which in turn leads to a terminating tower by Thomas' theorem.

Let $\pi_{\alpha,\beta}$ be the natural map from G_α to G_β.

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Proof of main theorem, continued

Let $H_\alpha = \{\, g \in G_\alpha \mid \exists\beta\, \pi_{\alpha,\beta}(g) = 1 \,\}$. Let β_g be least such that $\pi_{\alpha,\beta_g}(g) = 1$. Let

$$f(\alpha) = \sup_{g \in H_\alpha} \beta_g$$

Thus, f is a function from ordinals to ordinals. Every such function has a limit γ such that $\alpha < \gamma$ implies $f(\alpha) < \gamma$. I claim that G_γ has trivial center. If g is in the center of G_γ, then $\pi_{\gamma,\gamma+1}(g) = 1$. But $g = \pi_{\alpha,\gamma}(h)$ for some $h \in G_\alpha$, so

$$\pi_{\alpha,\gamma+1}(h) = \pi_{\gamma,\gamma+1}(\pi_{\alpha,\gamma}(h)) = \pi_{\gamma,\gamma+1}(g) = 1.$$

Consequently, $h \in H_\alpha$ and $\pi_{\alpha,f(\alpha)}(h) = 1$, so

$$g = \pi_{\alpha,\gamma}(h) = \pi_{f(\alpha),\gamma}(\pi_{\alpha,f(\alpha)}(h)) = \pi_{f(\alpha),\gamma}(1) = 1,$$

as desired. □

For centerless groups, Simon Thomas had provided an attractive bound on how long it takes the tower to terminate: the tower of a centerless group G terminates before $(2^{|G|})^+$. For groups in general, we ask

Question

How tall is the automorphism tower of a group G?

Unfortunately, the proof of the Main Theorem provides almost no information. We don't know how fast growing the function f can be.

Even when the initial group is finite, we have essentially no nice bound on the height of the tower.

Towers of Lie Algebras

The derivation tower of a lie algebra is obtained by iteratively computing the derivation algebra.

Enriqueta Rodríguez-Carrington has observed that the natural modification of my argument shows that the derivation tower of every Lie algebra eventually leads to a centerless Lie algebra. Since Simon Thomas has proved that the derivation tower of every centerless Lie algebra eventually terminates, we obtain:

Corollary

Every Lie algebra has a terminating derivation tower.

A False Hope

Since the automorphism tower kills off the center at each step, one might hope that G_ω is always centerless. But this is not so.

The dihedral group with eight elements has a center of size two, but is isomorphic to its own automorphism group (swap a and b in the presentation $\langle a, b \mid a^2 = 1, b^2 = 1, (ab)^4 = 1 \rangle$). The group at ω is the two element group, which still has a center, and so this tower survives until $\omega + 1$.

Simon Thomas has constructed finite groups whose towers have height $\omega + n$, but these also become centerless at stage $\omega + 1$.

We might focus on determining when the automorphism tower of a given group G becomes centerless. In particular, how large is this compared with $|G|$?

An Innocent Question

Question

Can you predict the height of the automorphism tower of a group G by looking at G?

You may argue, philosophically, that of course the answer is Yes, because the automorphism tower of a group G is completely determined by G; one simply iterates the automorphism group operation until the termination point is obtained, and that is where the tower terminates.

But nevertheless, I counter just as philosophically that the answer to the question is No! How can this be?

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A set-theoretic essence

The reason for my negative answer is that the automorphism tower of a group has a set-theoretic essence; building the automorphism tower by iteratively computing automorphism groups is rather like building the cumulative hierarchy V_α by iteratively computing power sets.

The fact is that the automorphism tower of a group can depend sensitively on the model of set theory in which you compute it. The very same group can lead to wildly different automorphism towers in different set theoretic universes.

Towers in Alternative Universes

Theorem (Hamkins+Thomas)

It is relatively consistent with ZFC *that there is a group G whose automorphism tower depends on the model of set theory in which it is computed.*

Specifically, we can construct a group G whose tower has any prescribed height α, such that for any β up to some preselected bound, there is another model of set theory where the tower of G has height β.

For the remainder of this talk, I would like to give a stratospheric view of the proof of this theorem.

What Do We Study?

- Group theorists study . . . . groups.
- Ring theorists study . . . . rings.
- Topologists study . . . . topological spaces.
- And so on . . . .
- Set theorists study . . . . models of set theory.

Each model of set theory is rich enough to form an entire mathematical universe. We can imagine living a full mathematical life entirely inside any one of these models.

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Mathematical Universes

Given a model of set theory, V, the method of *forcing* is used to construct a larger model of set theory, $V[G]$, something like constructing a field extension. One adds a “generic” object G and ensures that everything in the extension $V[G]$ is constructible from objects in the ground model V and the generic object G.

Paul Cohen, winning the Fields Medal for this result, used the method of forcing in 1962 to add many new subsets of ω, so that $V[G]$ violated the Continuum Hypothesis, because it had more than $\aleph_1$ many real numbers.

Forcing Affects Automorphisms

The method of forcing can be used to add new automorphisms of objects in the ground model.

For example, a mathematical object might be rigid in the ground model, having no nontrivial automorphisms there, but by forcing we can add a generic automorphism of this very same object, and therefore destroy its rigidity.

An uncountable object can even be made countable by forcing! One can add a generic bijection of the object with the natural numbers.

Generic Isomorphisms

To construct a group whose tower can be affected by forcing, the key set theoretic idea is the realization that forcing can make non-isomorphic structures isomorphic.

Warm-up Problem

Construct rigid non-isomorphic objects S and T which can be made isomorphic by forcing while remaining rigid.

The solution is to use generic Souslin trees. They are rigid and non-isomorphic, but can be forced isomorphic while preserving rigidity.

More Generally

Theorem (JDH and Thomas 1997)

One can add, for every regular cardinal κ, a set $\{ T_\alpha \mid \alpha < \kappa^+ \}$ of pair-wise non-isomorphic rigid trees such that for any equivalence relation E on κ^+, there is a forcing extension preserving rigidity in which the isomorphism relation on the trees is exactly E.

$\triangle \quad \square \quad \bigcirc \quad \Diamond \quad \oplus \quad \bowtie \quad \ominus$

These trees will be the unit elements of elaborate graphs whose automorphism groups we will precisely control by forcing.

The Normalizer Tower

Automorphism towers are too difficult to think about, so we work instead with the *normalizer tower* of a group. Given $G \leq H$, define

$$\begin{aligned} N_0(G) &= G \\ N_{\alpha+1}(G) &= N_H(N_\alpha(G)) \\ N_\lambda(G) &= \bigcup_{\alpha<\lambda} N_\alpha(G), \text{ if } \lambda \text{ is a limit} \end{aligned}$$

The Two Towers

Fact

The automorphism tower of a centerless group G is exactly the normalizer tower of G computed in the terminal group $H = G_\gamma$. That is, $G_\alpha = N_\alpha(G)$.

Fact

Conversely, if $H \leq \mathrm{Aut}(K)$, where K is a field, then by making a few modifications (adding a few bells and whistles) to the normalizer tower of H in $\mathrm{Aut}(K)$, one obtains an automorphism tower of the same height.

Thus, to make automorphism towers of a specific height, we need only make normalizer towers of that height in the automorphism group of a field.

Graphs Suffice

Theorem (Fried and Kollar 1981)

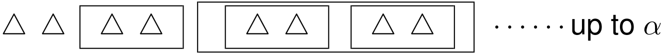
By adding points, any graph can be made into a field with the same automorphism group.

Thus, we can restrict our attention to subgroups of the automorphism groups of graphs. Since any tree can be represented as a graph, we make the connection with our earlier set-theoretic argument.

There, we obtained a delicate skill to make trees isomorphic while preserving their rigidity. By combining these trees in elaborate combinations, we will construct graphs whose automorphism groups we can precisely modify by forcing.

Constructing the Group

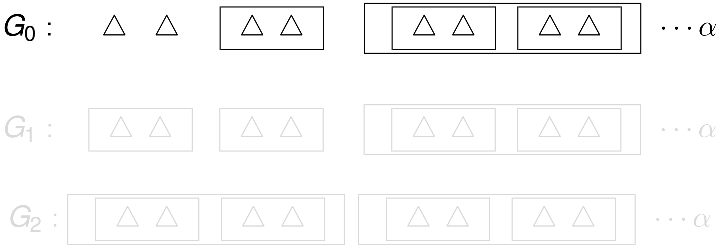
Let $\triangle$ be a rigid graph and build the following graph and subgroup of its automorphism group.



The intended subgroup, a large wreath product, is indicated by the boxes. Only those permutations which iteratively swap the components of any of the boxes are in the subgroup.

The Normalizer Tower

of this group is computed with ease.



And so on up to α

The normalizer tower of G grows for α many steps and then terminates. From this, we construct a centerless group whose automorphism tower has height α.

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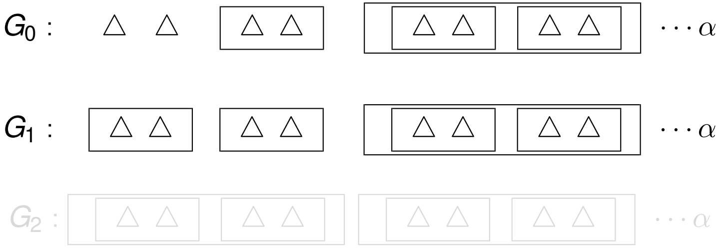

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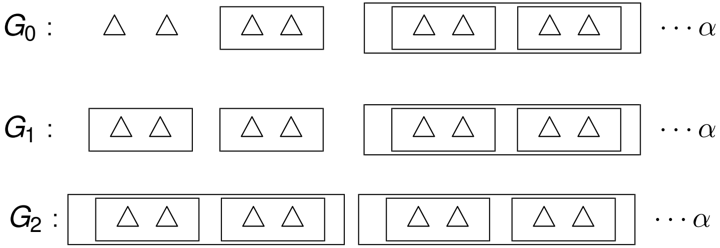

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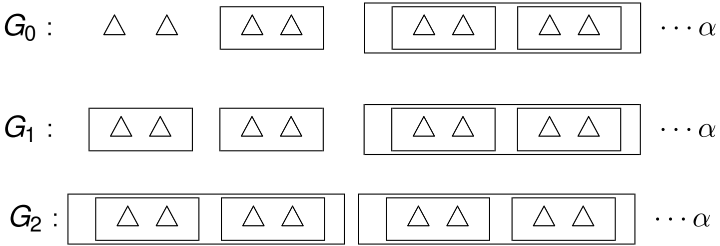

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The Exciting Twist

is to use the rigid non-isomorphic objects $\triangle$, $\square$, $\bigcirc$ and $\lozenge$ from the set-theoretic kernel.

$$G = \triangle \quad \square \quad \boxed{\bigcirc \quad \bigcirc} \quad \boxed{\boxed{\lozenge \quad \lozenge} \; \boxed{\lozenge \quad \lozenge}} \quad \cdots\cdots$$

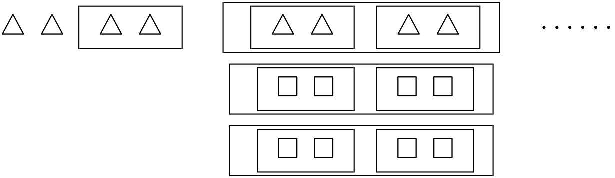
Forcing can ensure $\triangle \cong \square \cong \bigcirc \cong \lozenge \cong \cdots$ up to any given β, so in the forcing extension this is isomorphic to the original picture

$$G \cong \triangle \quad \triangle \quad \boxed{\triangle \quad \triangle} \quad \boxed{\boxed{\triangle \quad \triangle} \; \boxed{\triangle \quad \triangle}} \quad \cdots\cdots$$

In the forcing extension, therefore, the very same group has a tower of height β, as desired.

Forcing a Lower Height

One can also force the height of the tower to go down by activating a sort of wall which prevents the normalizer tower proceeding through:



By forcing $\square \cong \triangle$, it is easy to see that the normalizer tower will not continue past the wall.

Summarizing the argument

We described a group G in the automorphism group of graph. With the method of forcing, we can make precise changes to the automorphism group of this graph, in a larger model of set theory. In the resulting forcing extension, the normalizer tower of G, and hence also the corresponding automorphism tower, can be precisely controlled, so as to have any desired prescribed height.

The Conclusion

The automorphism tower of a group can have wildly different behavior in different models of set theory.

How Tall is the Tower of G?

Currently, the best bound on the height of the tower of an arbitrary group G is essentially the next inaccessible cardinal.

Such is our pitiful knowledge even in the case of finite groups!

Contrast this with the fact that the tallest towers we know of for finite groups have height $\omega + n$ for finite n. So the true answer lies somewhere between $\omega + n$ and the least inaccessible cardinal. How tall is the automorphism tower of a finite group?

Mutable groups in L

Simon Thomas and I created the mutable-tower groups by forcing to add them. We constructed the groups in a forcing extension of the universe, rather than in the universe itself. Thus, it remained open whether there are such groups in every model of ZFC or, for example, in the constructible universe L.

Gunter Fuchs and I answered this latter issue.

Theorem (Fuchs+Hamkins)

There are groups in L whose automorphism towers are mutable in forcing extensions, in the sense of the Hamkins-Thomas theorem.

We replace the generic Souslin-tree argument with highly rigid Souslin trees constructed from $\diamondsuit_\kappa$.

Additional set-theoretic dependence

Gunter Fuchs and Phillipp Lücke extended the phenomenon by showing that one may have a group G whose tower height can be controlled by forcing in a sequence of steps so as to realize any given (nonzero) ordinal pattern. Up, up, down, up, down, etc.

universe:	V	$V[G]$	$V[G * H]$	$V[G * H * I]$	$\cdots$
tower height:	0	$\omega + 1$	$\aleph_{\omega^2} + 2$	3	$\cdots$

That is, there can be a group G such that, by moving to successive forcing extensions, one can make the height of the tower go up and down so as to realize any prescribed pattern of ordinal heights.

Open Questions

- Is there a countable group with an uncountable automorphism tower?
- Is there a finite group with an uncountable automorphism tower?
- Is there a finite group G such that G_ω is infinite?
- For which γ is there a group whose tower becomes centerless in exactly γ many steps?
- Is there an infinite group G whose automorphism tower has height $(2^{|G|})^+$ or more?
- (Scott 1964) Is the finite part of the automorphism tower of every finite group eventually periodic?

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