CLASSIFICATION OF RESEARCH

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Abstract. This is a summary account by topic of the research publications of J. D. Hamkins.

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I would like to explain the major themes of my research program, which has pursed several lines of inquiry.

1. Forcing and large cardinals; indestructibility; infinite combinatorics:

Much of my work has focussed on the interaction of two central set-theoretic ideas, forcing and large cardinals. Motivated by the importance of understanding how our principal tools interact, I have sought to find out how large cardinals are or can be affected by forcing. This line of investigation leads naturally to the large cardinal indestructibility phenomenon, the lifting phenomenon, destructibility and superdestructibility, the Kunen inconsistency and its generalizations, particularly to forcing extensions and many other topics. [1] [2] [3] [4] [5] [6] [7] [8] [9] [10] [11] [12] [13] [14] [15] [16] [17] [18] [19] [20] [21] [22] [23].

2. Amenability of large cardinal embeddings to ground model:

The lifting phenomenon arises when the large cardinal embeddings $j : V[G] \to \bar{M}$ that exist in a forcing extension of the universe V are necessarily lifts of embeddings $j \upharpoonright V : V \to M \subseteq \bar{M}$ that exist already in the ground model. The fact that many of the commonly considered forcing extensions exhibit a robust lifting property has led to many applications. [24] [25] [26] [27].

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3. New large cardinal axioms

In several articles, I undertook an investigation of new large cardinal notions, including the tall cardinals, the unfoldable cardinals and the wholeness axiom. [28] [29] [30] [31].

4. Forcing; forcing axioms; generic absoluteness

Several of my projects have concerned new forcing axioms and generic absoluteness. [28] [32] [33] [34] [35].

5. The model theory of set theory

Some of my newer work can be classified as a part of the model theory of set theory, studying the model-theoretic features of the models of set theory. [36] [37] [38] [39] [40] [41] [42].

6. The modal logic of forcing, the multiverse and set-theoretic geology

In joint work, Benedikt Löwe and I introduced the investigation of the modal logic of forcing. This topic is connected with the maximality principle, a forcing axiom naturally expressed in terms of the forcing modalities. The study of the natural Kripke model of all forcing extensions of a given model leads naturally to the ideas of set-theoretic geology and the ground axiom. [43] [44] [45] [46] [47] [48] [49] [50] [51] [52] [53] [54] [55] [56].

7. Group theory/set theory; the automorphism tower problem

The automorphism tower of a group is the result of iteratively computing the automorphism group, mapping each group into the next by inner automorphisms: $G \to \mathrm{Aut}(G) \to \mathrm{Aut}(\mathrm{Aut}(G)) \to \cdots$ and so on, taking direct limits at limit stages. The question is whether there is a fixed point, a group that is isomorphic to its automorphism group by the natural map. Wielandt [57] proved classically that any finite centerless group has a terminating automorphism tower, and this phenomenon was gradually strengthened to centerless Černikov groups [58] and centerless polycyclic groups [59], until Simon Thomas [60], [61] proved that every centerless group has a terminating automorphism tower. Ultimately, I proved in [62] that every group has a terminating transfinite automorphism tower. Subsequently, Thomas and I showed in [63] that a single group can have wildly different automorphism towers in different models of set theory. In [6] [64], using strong notions of rigidity for Souslin trees, Fuchs and I showed that such groups exist under $\diamondsuit$. [64] [65] [63] [62].

8. Infinitary computability

I have made a major investigation of infinitary computation, introducing the model of infinite time Turing machines with Lewis in [66] in order to do so. The basic idea is to extend the operation of ordinary Turing machines into transfinite ordinal time, giving rise to an infinitary computability theory on the reals. This theory lies on the ample boundaries between computability theory and descriptive set theory, with increasing links to the much earlier work on higher recursion theory. The growing community of researchers in this area includes several prominent set theorists, as well as an enormous group of enthusiastic young researchers, many of whom are writing dissertations and masters theses on these new concepts of infinitary computability. Some of the most recent work builds connections between infinitary computability and descriptive set theory, particularly the theory of Borel equivalence relations. [67] [68] [69] [70] [71] [72] [73] [74] [75] [76] [77] [78] [79] [80] [81] [66].

9. Computability theory

Some of my other work concerns aspects of classical computability theory, including the computable analogue of Borel equivalence relation theory, where one studies computable reducibilities on equivalence relations of c.e. structures. [82] [83] [84] [85].

10. Infinitary game theory

I have undertaken some work on the theory of infinite chess, including work with Brumleve and Schlicht on the decidability of the mate-in-n problem of infinite chess and with C. D. A. Evans on the existence of transfinite game values in infinite chess. [82] [86].

11. Philosophy of set theory

I have become involved in the emerging debate on pluralism in the philosophy of set theory, defending a multiverse perspective in set-theoretic ontology. [44] [45] [43].

12. Infinitary utititarianism

With Barbara Montero and Donniell Fishkind, I have undertaken several projects on the topic of infinitary utilitarianism. [87] [88] [89].

13. (BOOK REVIEWS)

My three invited book reviews are: [90] [91] [92].

14. (ON TEACHING)

I wrote an account of the video-feedback method of teacher training that I ran at UC Berkeley mathematics department in 1995. [93]

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