Logic 2/4/15

Review:

Fact: every truth function (in any # of variables) ≡ an assertion in disjunctive normal form.

Example:

P	Q	R	$g(P, Q, R)$
T	T	T	F
T	T	F	F
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	F
F	F	F	T

Claim is that this pattern is producable by an expression in disjunctive normal form: Look where the T's are:

$$(\neg P \land \neg Q \land \neg R) \lor (\neg P \land Q \land R) \lor (P \land \neg Q \land R)$$

These pick out the rows in which the expression is true, and only those rows.

Can we use the same idea to prove this for conjunctive normal form?

You pick out all the <u>false</u> rows and each conjunctive clause will specify that you're <u>not</u> in each of the F rows.

So: $(\neg P \vee \neg Q \neg R) \wedge (\neg P \vee Q \vee R) \wedge$
$(\neg P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee Q \vee \neg R)$

(so for D.N.F, you say that you're in one of the truth rows or another. for C.N.F. you say that you're in none of the F rows.)

HW Review

$\boxed{7a}$ two options:

1) let's see what truth functions we can get w/ $\leftrightarrow$ & $\neg$.

P	Q	$\neg P$	$\neg Q$	$P \leftrightarrow Q$	$P \leftrightarrow P$	$P \leftrightarrow \neg P$	$\neg(P \leftrightarrow Q)$
T	T	F	F	T	T	F	F
T	F	F	T	T	T	F	F
F	T	T	F	F	T	F	F
F	F	T	T	F	T	F	T

$P \leftrightarrow (P \leftrightarrow P)$ $\quad P \leftrightarrow \neg(P \leftrightarrow Q)$ $\quad (P \leftrightarrow \neg P)$ $\quad (P \rightarrow \neg Q)$

If you "iff" any two of these columns, are you still on the list? if you $\neg$ any of these columns, are you still on the list?

Yes and Yes. $\therefore$ $\{\leftrightarrow, \neg\}$ is not complete b/c it can't express truth functions not on the list.

Q: do you have to check manually?

argument option 2)

P	Q	$\neg P$	$P \leftrightarrow Q$
T	T	F	T
T	F	F	F
F	T	T	F
F	F	T	T

notice that $\neg P$ & $P \leftrightarrow Q$ always have an even # of Ts. The property of having an even # of Ts is preserved by $\neg$. let's prove: if φ and ψ both have an even # of Ts, then $\varphi \leftrightarrow \psi$ also has an even # of Ts. (this would prove $\{\neg, \leftrightarrow\}$ incomplete)

. . . . .

an even # of rows

an even # of rows

φ	ψ	$\varphi \leftrightarrow \psi$
$\boxed{T}$	$\boxed{F}$	F
	$\boxed{T}$	T
		F
$\boxed{F}$	$\boxed{F}$	T

IF "TT" is even, "TF" is even, "FT", and "FF" are even, so $\varphi \leftrightarrow \psi$ has an even # of Ts.

IF 'TT' is odd, 'TF' is odd & 'FT' is odd and 'FF' is odd, so $\varphi \leftrightarrow \psi$ has an odd # of Ts.

$\therefore$ anything we can build w/ $\leftrightarrow$ & $\neg$ has an even # of Ts; $\therefore$ $\{\leftrightarrow, \neg\}$ incomplete.

Multi-valued logic

Łukasiewicz introduced the idea of using more than just T and F.

lets introduce a third value, U. This called be intepreted in a variety of [incorrect] ways, but whats impotant is that its distinct from T & F (for now).

P	Q	¬P	P ∧ Q	P ∨ Q	P → Q (~~Kleene~~ ¬P ∨ Q)	P → Q (Łuka logic)
T	T	F	T	T	T	
T	U	F	U	T	U	
T	F	F	F	T	F	
U	T	U	U	T	T	U
U	U	U	U	U	U	
U	F	U	F	U	U	
F	T	T	F	T	T	
F	U	T	F	U	T	
F	F	T	F	F	T	

Łuk logic →

negation of something unknown is also unknown (though we could say whatever we want?)

There are different logics in 3-value logic
We could define P → Q the classical way, as ¬P ∨ Q.
But Łuk. thought U → T should be U, for some (?) reason.

Kleene ~~'s~~ logic is used in lots of CS. U is taken in those cases as "Missing information" - allows the database to reason with missing info.

But is it really the logic of Missing information? This implies that there is a true state of affairs that we just don't know.

let's Suppose (a, b, c) is a list of tvk truth values (e.g. (T,T,U)). A list is crisp if all values are classical.

Thus a crispification of a row in a truth table is a crisp list that turns U into either ~~T~~ Ts or Fs. There are two crispifications for each row.

Question: if f is an expression & $f(a_1, \ldots a_n) = U$ (i.e. that row of the truth table is U), is there a crispification to $(a_1^*\ a_2^* \ldots a_n^*)$ such that $f(a_1^* \ldots a_n^*) = T$ and another $f(a_1^{**} \ldots a_n^{**}) = F$

In other words, is it the case that if there is a U, it could have been either a T or a F, but we need more information to settle the matter?

Ex 1: give a counter example - a wwf which comes out U, but for which there are not two configurations making it T and F. (hint: 1 variable)

Because of this, SQL logic will give unexpected results when used to reason w/ missing information.

Q: Is tvk logic the logic of possible worlds? We have many possible worlds, and T means T in all worlds, F means false in all worlds, and U means T in some, F in others.

NO: $P \lor \neg P$ is True in all possible worlds, we think. But tvk gives you U's for $P \lor \neg P$'s truth table, which would suggest that there are worlds where $P \lor \neg P$ is false, and thats obviously not right.

w/ 3 values, There are 3^9 possible truth functions.

Logic of missing information & logic of possible worlds are not truth functional [?]

Are there any tautologies in tuk logic?

Strong (classical) tautology: all Ts in T-Table

are there any strong tautologies?

$P \to P$? no, b/c it has a Us for $U \to U$.

$P \leftrightarrow P$? Well, it's natural to define $(P \leftrightarrow Q)$ as $(P \to Q) \wedge (Q \to P)$. If we define it that way, $U \leftrightarrow U \equiv (U \to U) \wedge (U \to U) \equiv U \wedge U \equiv U$, So no, its not a strong tautology.

Fact: There are no strong tautologies in tvk logic using $\neg, \wedge, \vee, \leftrightarrow$, and $\to$. There is a truth function w/ all Ts, but you can't build it with our 5 connectives. This is because They are all U-preserving (you can never get a T w/ these connectives if all you have are U inputs)

[But there are strong tautologies in ±VK logic — we just can't get them w/ our fave five] right?
(?)

So $\wedge \vee \neg \leftrightarrow \rightarrow$ are not strongly, classically complete in ±VK logic.

"weakly complete" ⇒ you can express anything in the language using only $\wedge \vee \neg \leftrightarrow \rightarrow$. (why do we care about this and give it a name?)
O.H. (?)

$\boxed{\text{Ex 2}}$ Define a natural version of $\uparrow$ & $\downarrow$ in ±VK logic → does it obey the expected equivilences? Are they strongly complete? Are they "weakly" complete?

$\boxed{\text{Ex 3}}$ Verify the DeMorgan's laws in ±VK.

$\varphi \equiv \psi$ still means φ and ψ have the same truth table. In ±VK, is this still the same as "$\varphi \leftrightarrow \psi$ is a tautology"

Probably not! Because there aren't any tautologies using just $\leftrightarrow, \neg, \rightarrow, \wedge, \vee$, and there are probably

~~[illegible]~~

quite a few ψs $\equiv$ φs.

E.g. $P \equiv P$, but $P \leftrightarrow P$ not a taut.

Q.# Q: how did we know ahead of time that there were an ∞ # of φs $\equiv$ to ψs?

$\boxed{\text{Ex 4}}$ verify distributivity laws

$A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$

$A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C)$

Same as "expression"?

Claim: in tvk, any wwf built from $\neg, \leftrightarrow, \rightarrow, \wedge, \vee$ is $\equiv$ to an expression in D. N. F. Argument: (works for classical logic too)

Suppose we have a φ built from $\neg, \wedge, \vee, \rightarrow$, and $\leftrightarrow$. We can eliminate $\rightarrow, \leftrightarrow$, because we can express them with $\neg$, $\vee$, & $\wedge$.

Mixing spatial metaphors!

W/ Demorgan's laws, we can push all $\neg$s into lower levels of the "tree", making all occurrences of $\neg$s in literals.

$\neg(A \wedge B) \equiv \neg A \vee \neg B$; $\neg(A \vee B) \equiv \neg A \wedge \neg B$

Using first distributive law: $A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$, we can put all $\wedge$s inside / "under" $\vee$s. Gives you D.N.F at the end.

~~[illegible]~~

Q: IF $\varphi \equiv \ell$ classically, are they $\equiv$ in ŁUK logic? NO!

P	Q	$P \vee \neg P$	$(P \vee \neg P) \wedge (\underline{Q \vee \neg Q})$	
T	T	T	T	T
T	M	T	U	U
T	F	T	T	T
U	T	U	U	T
U	U	U	U	U
U	F	U	U	T
F	T	T	T	T
F	U	T	U	U
F	F	T	T	T

different.

$\boxed{Ex5}$ Find an expression using P, Q & R which is U if any of them is U and otherwise is F. [hint: it's gonna be a negation of a classical taut, since it needs to be F on all classical rows.]

$\boxed{Ex6}$ is the T-table of ℓ (using only $\neg, \vee, \wedge, \rightarrow, \& \leftrightarrow$) in ŁUK determined by the location of the T's?
[hint: Find a counterexample - 2 wffs w/ same Ts but diff Us + Fs]

wffs built from ∨ ∧ ¬ → ⇒ "the standard language"

"weak tautologies"

"Pair consistent" logic → U is both T & F, so things at U are T. U is a "designated truth value" it's true and also false. So a weak taut is a wff in which every row is either T or U.

Theorem: if φ is expressed using $\neg, \vee, \wedge, \rightarrow, \leftrightarrow$, then φ is a weak tautology in LVK iff φ is a classical tautology. Proof:

OH (1) assume φ is a weak tautology - every row either T or U in LVK T-table. Every classical row has a classical value. But the only classical value is T, so φ is a classical tautology

How do we know it's not F? ?

(2) assume φ is a classical taut. φ is in D.N.F. Suppose φ has a row in the LVK w/ an F. So on that row, every ∧ clause is F. So at least one conjunct in each ∧ clause is F (U doesn't work). So the classical values of the inputs on this row are making the φ false. But we said φ was a classical tautology, ⊥.

∴ There is no F on φ's LVK truth table, weak taut.