I know that you know that I know that you know...

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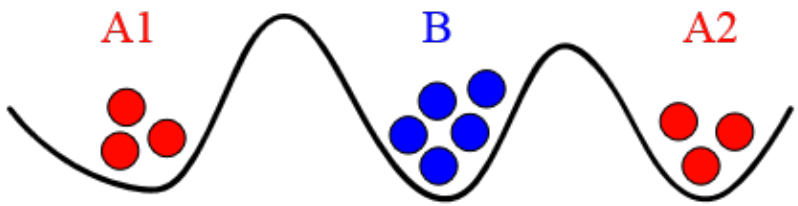
Epistemic Logic puzzles

I was very pleased to discover that the theme of this conference is Knowledge:

So please allow me to show you some fun puzzles in the logic of knowledge.

The two generals problem

Two red generals want to coordinate their attack on the blue army, but are separated by hills. If both attack, it will succeed. But if only one general attacks, they will fail. Communication is difficult.



The first red general dispatches a messenger through the dangerous blue valley: *We attack at dawn; agreed?*

The message gets through! The other red general replies: *Yes, at dawn! Please confirm.*

Confirming the message

- A1: *We attack at dawn; agreed?*
- A2: *Yes, at dawn! Please confirm.*
- A1: *Received your message! Ready to go at dawn...provided we know you get this message.*
- A2: *Got it! We're definitely on, once we know you have received this.*

At each step, the generals seem to need confirmation that their message was received, in order that they would know that the other general is confident in the plan. Otherwise, each general would worry that he might be attacking alone.

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How much confirmation suffices?

Can the generals ever be confident?

Obviously, the first message requires confirmation.

Suppose that n is the smallest number of confirming messages that need to be sent.

But in this case, the protocol should work whether or not the last message is actually sent, since it might have been lost.

So $n - 1$ messages actually suffice, a contradiction.

Conclusion: no finite number of message can ensure that the generals have the information that is required to be confident in the plan.

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Another version: two parents

My wife usually picks up my daughter Hypatia, and I pick up my son Horatio. One day, it was convenient to swap.

I email her: *I'll pick up Hypatia today; you get Horatio. Please confirm; otherwise it is as usual.*

She texts: *Let's do it! Let me know, so I know we're on.*

I leave voicemail: *OK, we're on for the swap!...as long as I know you get this message.*

She emails: *Got the message. We're on! But let me know that you get this message so I can count on you.*

And so on...

At no stage can we know for certain that the other person has the necessary information.

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My wife usually picks up my daughter Hypatia, and I pick up my son Horatio. One day, it was convenient to swap.

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This is not just a silly problem!

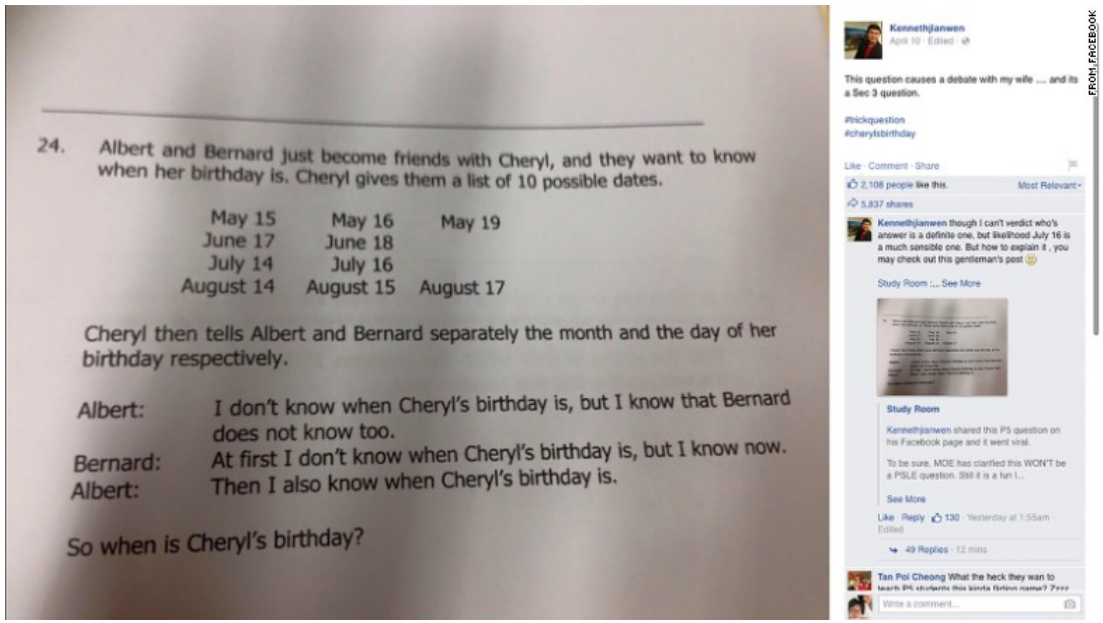
The issue arises in the design of internet protocols.

Two computers want to coordinate a data transfer—when will a video stream be sent? How will a financial transaction be completed? How can the two parties achieve the state of common knowledge that appears required?

General conclusion: they can't. protocols cannot be designed to achieve perfect coordination with common knowledge.

Cheryl's birthday

Recently a logic puzzle from the Singapore Math Olympiad went viral on the internet:



24. Albert and Bernard just become friends with Cheryl, and they want to know when her birthday is. Cheryl gives them a list of 10 possible dates.

May 15	May 16	May 19
June 17	June 18	
July 14	July 16	
August 14	August 15	August 17

Cheryl then tells Albert and Bernard separately the month and the day of her birthday respectively.

Albert: I don't know when Cheryl's birthday is, but I know that Bernard does not know too.

Bernard: At first I don't know when Cheryl's birthday is, but I know now.

Albert: Then I also know when Cheryl's birthday is.

So when is Cheryl's birthday?

The problem stumped millions of people all over the world!

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Both of them know this.

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So the month is not August. It must be July 16!

I don't know

Cheryl privately gives Albert and Bernard each a number, amongst $0, 1, 2, \ldots$

Cheryl: *You have different numbers. Whose is larger?*

Albert: *I don't know.*

Bernard: *I don't know either.*

Albert: *I still don't know whose number is larger.*

Bernard: *Alas, I remain in ignorance.*

Albert: *Ah, now that you say that, suddenly I know whose number is larger!*

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For Bernard to know that, he must have 4.

Albert: *And now I also know both numbers.*

Let's solve it. Albert and Bernard have different numbers, amongst $0, 1, 2, \ldots$ Whose number is larger?

Albert: *I don't know.*
So Albert doesn't have 0*.*

Bernard: *I don't know either.*
So Bernard doesn't have 0. He also doesn't have 1.

Albert: *I still don't know whose number is larger.*
So Albert does not have 1 *or* 2*.*

Bernard: *Alas, I remain in ignorance.*
So Bernard does not have 2 or 3.

Albert: *Ah, now that you say that, suddenly I know whose number is larger!*
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For Bernard to know that, he must have 4. And so Albert has 3.

Albert: *And now I also know both numbers.*
Albert can reason just as we did.

The blue-eyed islanders

There is a remote island with 100 perfectly logical inhabitants, all with blue-eyes.

In their cultural practice, they do not discuss eye color.

Indeed, if any of them should come to know they have blue eyes, they must leave the island the next dawn with a flashy display.

One day a trusted visitor arrives. Departing at the end of his visit, he says, *"At least one of you has blue eyes."*

Exactly one hundred days later, all the islanders make a big flashy display at dawn and everyone leaves the island.

Why?

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Why?

Three logicians walk into a bar

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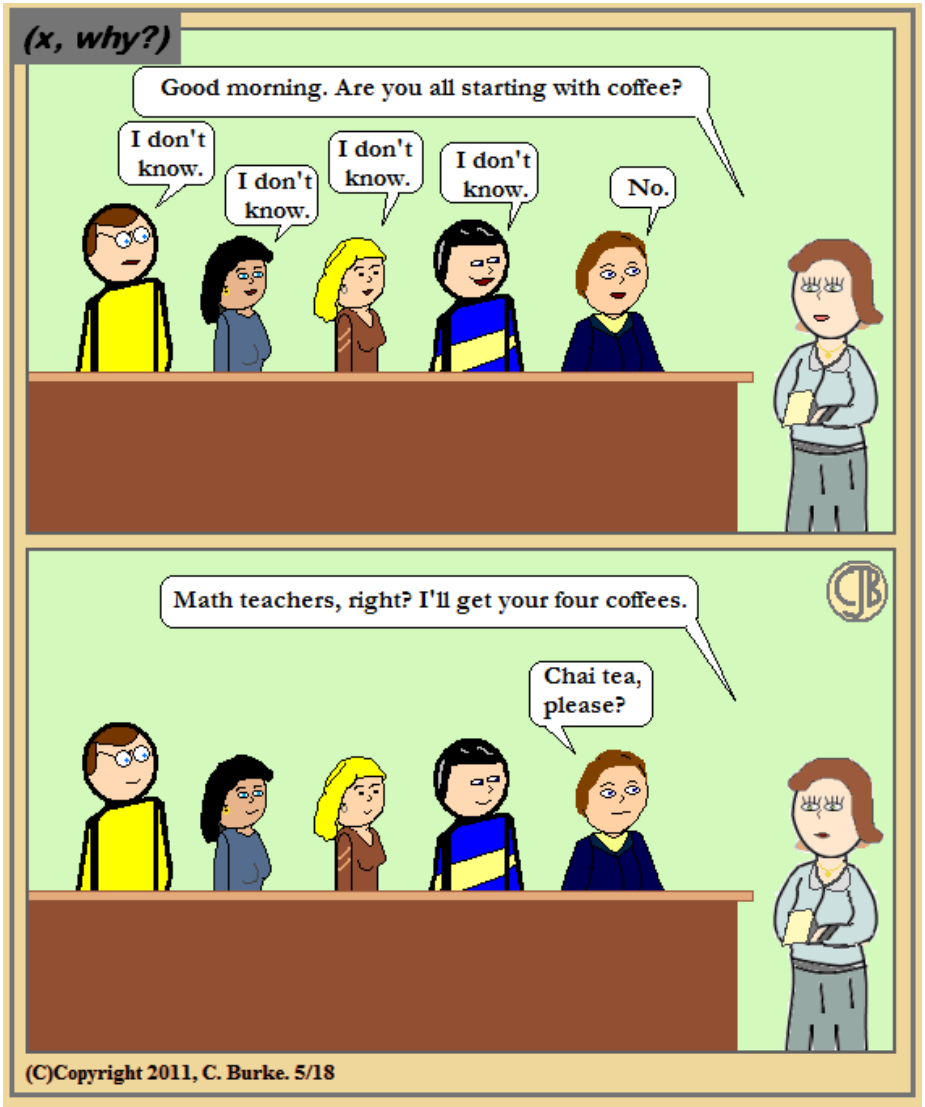
The bartender asks: *Do you all want beer?*

The first logician says: *I don't know.*

The second logician says: *I don't know.*

The third logician says: *Yes!*


(x, why?)
Good morning. Are you all starting with coffee?
I don't know.
I don't know.
I don't know.
I don't know.
No.
Math teachers, right? I'll get your four coffees.
Chai tea, please?
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Thank you.

Find more epistemic logic puzzles on my blog:

jdh.hamkins.org.

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The City University of New York