

FAILING DEFINITE DESCRIPTIONS

FOOD FOR THOUGHT SEMINAR, NOTRE DAME
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DEFINITE DESCRIPTIONS

THE BADLY JUGGLING CLOWN
THE KING OF FRANCE

RUSSELL/WHITEHEAD $\exists x \varphi(x) =$ "THE x SUCH THAT $\varphi(x)$ "

IMPLICIT COMMITMENT TO REFERENCE TO USE $\exists x \varphi(x)$ IMPLIES THERE IS A UNIQUE x $\varphi(x)$.

EXAMPLES

THE KING OF FRANCE IS BALD. $B(\exists x Kx)$
EVERYONE'S ^{PET} ELEPHANT IS GRAY.

N. THE LARGEST PRIME NUMBER IS ODD. $O(\exists x Px)$

R. $\forall x (x > 0 \rightarrow \frac{1}{x} > 0)$.

GOAL EXTEND FOL WITH SEMANTICS FOR 2 EXPRESSIONS.

STRONG SEMANTICS

ATOMIC ASSEPTIONS $A(t_1, \dots, t_n)$ TRUE WHEN ALL REFERENCES SUCCEED & A HOLDS.

EXTEND COMPOSITIONALLY TO $\varphi \vee \psi$, $\neg \varphi$, $\forall x \varphi(x)$ ETC.

RUSSELL + TARSKI

[APPLY TO EXAMPLES ABOVE.

CRITICISMS.

① DOES NOT FULFILL IMPLICIT COMMITMENT TO REFERENCE.
 $\neg B(\exists x Kx)$ TRUE.

TENSION RUSSELL VS. TARSKI

② DOES NOT RESPECT STIPULATIVE DEFNS.

N $O(\exists x Px)$ FALSE.

BUT IF DEFINE $O(x) = \neg \exists y y + y = x$, THEN TRUE.

③ SENSITIVITY TO WHAT COUNTS AS ATOMIC.

④ D/N RESPECT LOGICAL EQUIVALENCE.

$ODD_0(x) = \neg \exists y y + y = x$

$ODD_1(x) = \exists y y + y + 1 = x$

$\neg \text{NE } ODD_0(x) \leftrightarrow ODD_1(x)$

BUT $ODD_0(\exists x Px)$ TRUE
 $ODD_1(\exists x Px)$ FALSE

⑤ D/N RESPECT INSTANTIATION

$\forall x x = x$ T BUT $(\exists x Px) = (\exists x Px)$ F

WEAK SEMANTICS

- DISTINGUISH BETWEEN T, F, NOT MEANINGFUL.
- ASSERTIONS WITH FAILED REFERENCES ARE NOT MEANINGFUL.
- ATOMIC CASE: MEANINGFUL WHEN REFS SUCCEED. THEN T/F ACCORDINGLY.

$\varphi \wedge \psi$

$\neg \varphi$

$\forall x \varphi(x)$

IF ANY COMPONENT IS NOT MEANINGFUL, CORRUPTS WHOLE THING.

FACT IF φ IS MEANINGFUL, TRUTH VALUE IS SAME AS STRONG SEMANTICS.

NOTE LEM FAILS. $\varphi \quad \neg \varphi$ BOTH NOT MEANINGFUL. SO $\varphi \vee \neg \varphi$ NOT TRUE.
BUT IT'S NOT INTUITIONISTIC LOGIC. $\varphi \rightarrow \varphi$ ALSO FAILS.

NATURAL SEMANTICS

LESS HESITANT

ALLOW $\varphi \vee \psi$ TO BE T, IF ONE OF THEM IS. eg. BLARGH \vee 2, SPINE

$\varphi \wedge \psi$ F IS ONE IS F.

$\exists x \varphi(x)$

$\forall x \varphi(x)$

FACT NATURAL SEMANTICS = TRUTH VIA KLEENE LOGIC $\{T, F, \#\}$.

ADVANTAGES BOTH WEAK & NATURAL SEMANTICS:

- ① FULFILL IMPLICIT COMMITMENT TO REFERENCE
- ② STIP DEF
- ③ LOGIC EQ.
- ④ INSTANTIATION

FOR MEANINGFUL ASSERTIONS.

• FINAL DEFLATIONARY POINT

NONE OF THE 3 SEMANTICS HAVE ANY NEW EXPRESSIVE POWER.

FOR ANY φ IN LANG. WITH \perp

WE CAN EXPRESS THE TRUTH VALUES, MEANINGFULNESS IN LANGUAGE W/O \perp .

NOTHING AT STAKE. IT IS A QUESTION OF CONVENIENCE.