DEFINITE DESCRIPTIONS

THE BADLY JUGGLING CLOWN
THE KING OF FRANCE

RUSSELL/WHITEHEAD \( \exists x \, p(x) = \text{"the } x \text{ such that } p(x)" \)

IMPLICIT COMMITMENT TO REFERENCE TO USE \( \exists x \, p(x) \) IMPLIES THERE IS A UNIQUE \( x \) \( p(x) \).

EXAMPLES

THE KING OF FRANCE IS BALD. \( B(\exists x \, kx) \)
EVERYONE'S FAVORITE ELEPHANT IS GRAY. \( (\forall x \, \neg p(x)) \)
THE LARGEST PRIME NUMBER IS ODD. \( \neg (\exists x \, p(x)) \)
IN \(\forall x \, (x > 0 \rightarrow \frac{1}{x} > 0) \).

GOAL EXTEND FOL WITH SEMANTICS FOR \( \exists \) EXPRESSIONS.

STRONG SEMANTICS

ATOMIC ASSERTIONS \( A(t_1, \ldots, t_n) \) TRUE WHEN ALL REFERENCES SUCCEED & ARE.
EXTEND COMPOSITIONALLY TO \( \forall A, \neg A, \forall x \, \neg p(x) \) ETC.
RUSSELL & TARSKI

APPLY TO EXAMPLES ABOVE.

CRITICISMS.

1. DOES NOT FULFILL IMPLICIT COMMITMENT TO REFERENCE.
   \( \neg B(\exists x \, kx) \) TRUE.
   TENSION RUSSELL VS. TARSKI

2. DOES NOT RESPECT STIPULATIVE DEFNS.
   IN \( \neg (\exists x \, p(x)) \) FALSE.
   BUT IF DEFINE \( (\exists x) = \exists y \, y + y = x \), THEN TRUE.

3. SENSITIVITY TO WHAT COUNTS AS ATOMIC.

4. \% RESPECT LOGICAL EQUIVALENCE.
   \( \neg \exists \, \forall (x) = \exists y \, y + y = x \) \( \neg \exists \, \exists (x) \rightarrow \forall (x) \).
   BUT \( \exists \, (\exists x \, p(x)) \) TRUE
   \( \exists \, (\exists x \, p(x)) \) FALSE

5. \% RESPECT INSTANTIATION

\( \forall x \, x = x \) BUT \( (\exists x \, p(x)) = (\exists x \, p(x)) \)
\( \top \) \( \bot \)
WEAK SEMANTICS

DISTINGUISH BETWEEN T, F, NOT MEANINGFUL.

ASSERTIONS WITH FAILED REFERENCES ARE NOT MEANINGFUL.

ATOMIC CASE: MEANINGFUL WHEN REF Succeed, THEN T/F ACCORDINGLY.

∀x P(x)   IF ANY COMPONENT IS NOT MEANINGFUL, CORRUPTS WHOLE THING.

FACT IF P IS MEANINGFUL, TRUTH VALUE IS SAME AS STRONG SEMANTICS.

NOTE LEM FAILS. ∃x P(x) BOTH NOT MEANINGFUL. SO ∃x P(x) NOT TRUE.

BUT IT'S NOT INTUITIONISTIC LOGIC. P → Q ALSO FAILS.

NATURAL SEMANTICS

LESS HESITANT

ALLOW ∀y ∀x to be T if one of them is. E.g., BLARGH ← WHITE

∀x F IS ONE IS T.

∃x ∃y P(y)

∀x ∃y P(y)

FACT NATURAL SEMANTICS = TRUTH VIA KLEENE LOGIC \{T, F, #\}.

ADVANTAGES BOTH WEAK & NATURAL SEMANTICS:

1. FULLFILL IMPLICIT COMMITMENT TO REFERENCE
2. STRIP DEF
3. LOGIC EQ.
4. INSTANTIATION

FINAL DEFLATIONARY POINT

NONE OF THE 3 SEMANTICS HAVE ANY NEW EXPRESSIVE POWER.

FOR ANY P IN LANG. WITH L
WE CAN EXPRESS THE TRUTH VALUES, MEANINGFULNESS
IN LANGUAGE W/O L.

NOTHING AT STAKE. IT IS A QUESTION OF CONVENIENCE.