What if your potentialism is implicitly actualist?

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Modal understanding of potentialism

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Fundamental aspects of various potentialist conceptions are often revealed in the modally expressible features.

For any given potentialist conception, we should seek to find exactly the modal logic that is expressed.

Diverse potentialist conceptions

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- Many different kinds of arithmetic potentialism
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- More general forms of potentialism, modal model theory, potentialist category theory

The various potentialist conceptions express subtly different philosophical ideas about the nature of potentialism and often have importantly different modal features.

My thesis

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- Meanwhile, other forms of potentialism are not like this.

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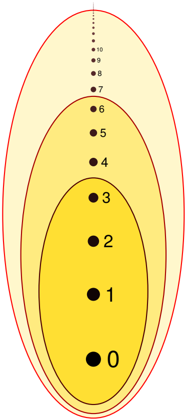
- In these cases, the actualist and potentialist perspectives are bi-interpretable.
- Consequently, for these forms of potentialism, there is little at stake in the actualism/potentialism dispute—they are simply two ways of looking at the same subject matter.
- Meanwhile, other forms of potentialism are not like this.
- The central distinction in potentialism should therefore become: convergent potentialism vs. divergent.

Initial-segment potentialism in arithmetic



The classic potentialist conception

Aristotelian potentialism

Possible worlds consist of all numbers up to some n.

$$u = \{\, 0, 1, 2, \ldots, n \,\}$$

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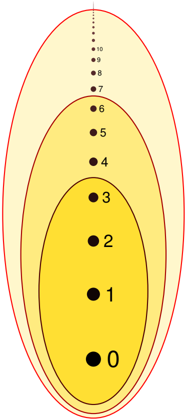

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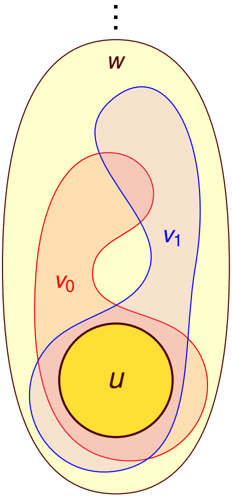

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Possible worlds = any finite set of numbers

Large numbers may come into actuality before some smaller numbers.

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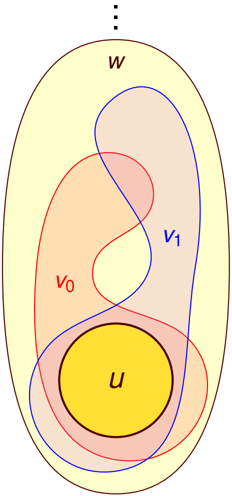

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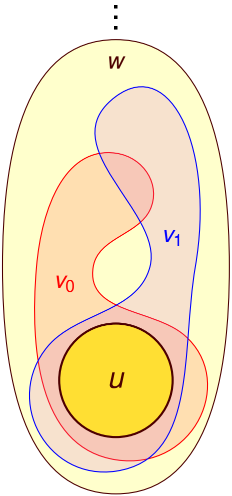

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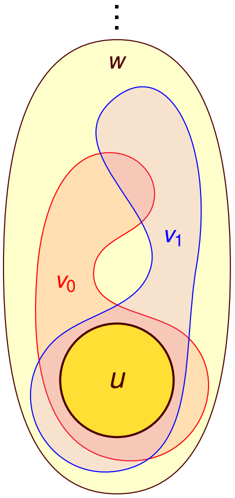

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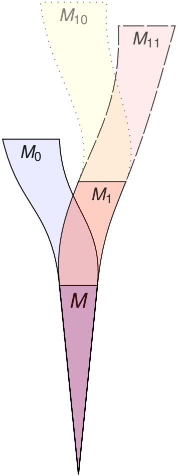

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End-extensional potentialism, for example, expresses a potentialist conception of *realms of feasibility*—speaks to ultrafinitism.

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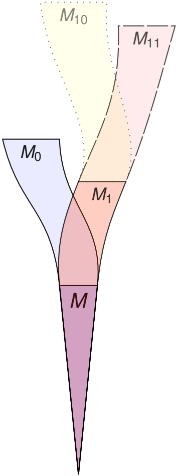

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Theorem (Hamkins [Ham18])

The modal validities of arithmetic end-extensional potentialism are exactly S4.

The proof makes use of the universal algorithm.

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Every world accesses its end extensions $M \sqsubseteq N$.
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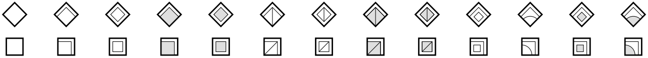
And many more, [Ham18].

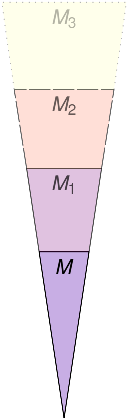


Enormous range of set-theoretic potentialism

Similarly we have numerous conceptions of set-theoretic potentialism.

Let me quickly mention several.

Set-theoretic rank potentialism



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A set-theoretic analogue of Aristotelian potentialism.

Perhaps the canonical example of height potentialism

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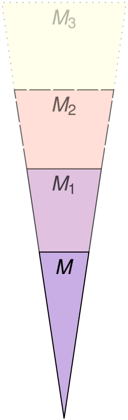

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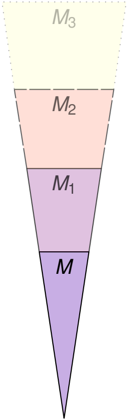

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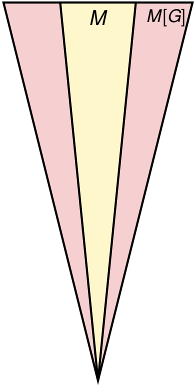
Modal validities = S4.3.

S5 is valid at V_κ for $\mathcal{L}_\in$ iff κ is Σ_3-correct

S5 is valid at V_κ for $\mathcal{L}_\in^\Diamond$ iff κ is fully correct.

Forcing potentialism

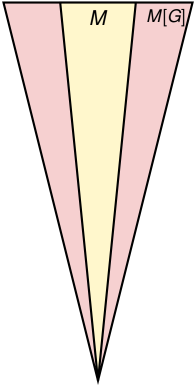


Worlds = models of set theory

Every world M accesses its forcing extensions $M[G]$.

Canonical example of width potentialism.

Forcing potentialism



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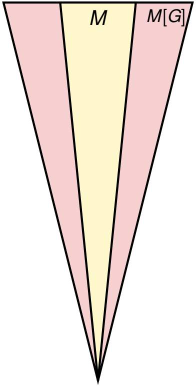

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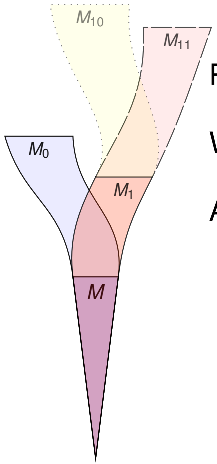
(Hamkins+Löwe [HL08]) The modal logic of forcing is exactly S4.2.

(Hamkins [Ham03]) And yet, also consistent that a model validates S5 for sentences

Top-extensional potentialism



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World M accesses its top extensions, $M \sqsubseteq N$.

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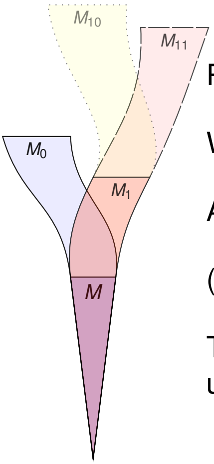

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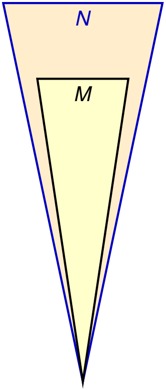
Set-theoretic end-extensional potentialism



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Each world accesses its end-extensions $M \sqsubseteq N$, which means new sets appear, but no set gains new elements.

A hybrid height/width potentialism

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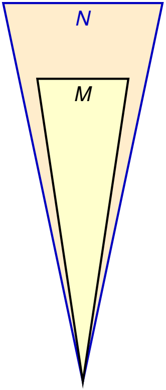

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A hybrid height/width potentialism

(Hamkins+Williams [HW21]) Modal validities are exactly S4.

There is divergent branching in this conception.

The proof uses a set-theoretic analogue of the universal algorithm: the universal Σ_1-definition.

Submodel potentialism

Possible worlds = models of set theory

World M accesses N when $\langle M, \in^M \rangle$ is a submodel of $\langle N, \in^N \rangle$.

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(Hamkins [Ham13; HL22]) The modal validities are a surprise: exactly S4.3

Potentialist systems

A *potentialist system* is:

- A collection $\mathcal{W}$ of structures M in a common language $\mathcal{L}$
- a reflexive transitive relation on these structures $\sqsubseteq$
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Alternative approach: $U \sqsubseteq W$ means there is an embedding $U \hookrightarrow W$. A system of counterparts.

A subtle point about accessibility

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Latter case leads to distinction between convergence & amalgamation

In my view, we would benefit from greater philosophical analysis of this distinction for potentialism.

Direct extension vs. embedding access

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Good news [HW24]: modal assertions in $\mathrm{Mod}(T)$ are the same!

In fact [AD22]: the two Kripke structures are bisimilar.

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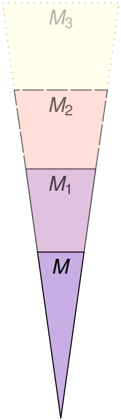
The limit structure may have a fundamentally different nature than the possible worlds.

Linear inevitabilism



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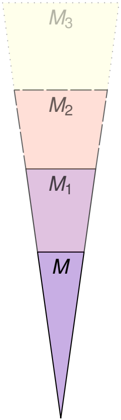

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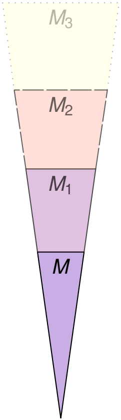

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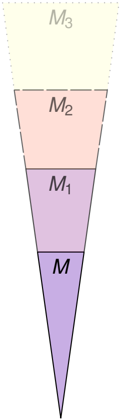

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But also:

$$\Diamond\varphi \wedge \Diamond\psi \to [\Diamond(\varphi \wedge \Diamond\psi) \vee \Diamond(\psi \wedge \Diamond\varphi)]$$

S4.3 is valid, and often exactly S4.3 is valid.

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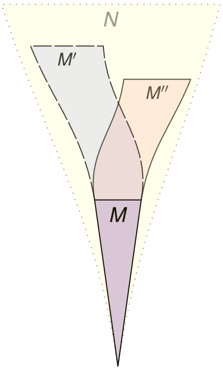

Other forms of potentialism are not linear, but nevertheless convergent.

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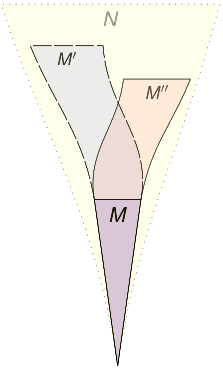

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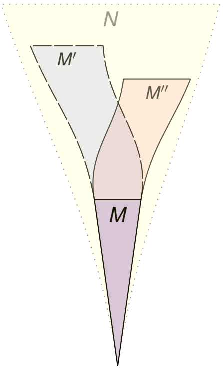

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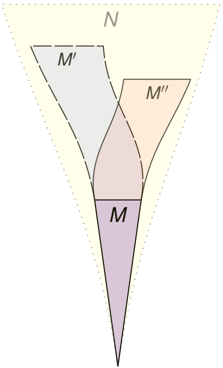

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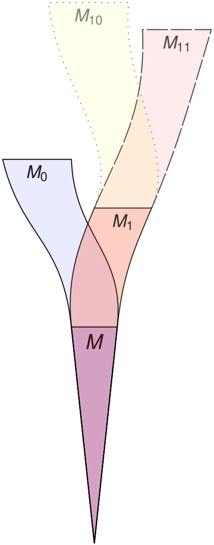
S4.2 is valid, and often exactly S4.2.

Divergent potentialism



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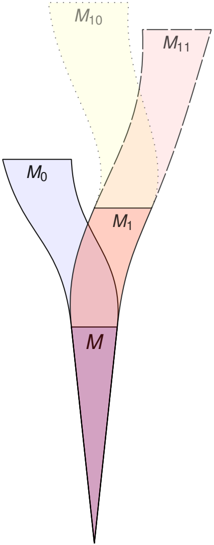

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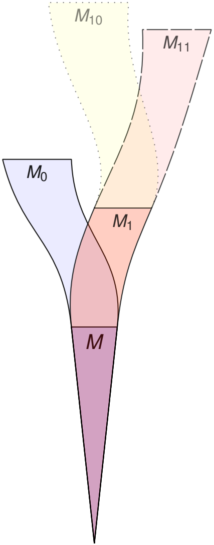

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Validates S4, and often exactly only S4.

(Height v. width) versus (convergence v. divergence)

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Forcing potentialism in the general case. Non-amalgamation phenomenon: extensions $M[c]$, $M[d]$ with no common forcing extension. (And yet, S4.2 is valid.)

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Theorem [HL22]

With convergent potentialism, if a potentialist system $\mathcal{W}$ of possible worlds converges to limit model M, then

$$M \models \psi \quad \text{if and only if} \quad W \models_{\mathcal{W}} \psi^\Diamond$$

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$$M \models \psi \quad \text{if and only if} \quad W \models_{\mathcal{W}} \psi^{\Diamond}$$

Thus, actualist truth in the limit model is expressible within potentialism.

Convergent potentialism is implicitly actualist

Using only the potentialist ontology, therefore, the convergent potentialist has a full accounting of:

- actualist objects
- actualist structure
- actualist truth, via the potentialist translation

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- actualist truth, via the potentialist translation

The potentialist denies the limit model exists, yet seems to know everything about it.

Implicit actualism

The convergent potentialist can thus give a completely clear account of the actualist model—like interpreting $\mathbb{C}$ in $\mathbb{R}$.

The actualist model is interpretable in the potentialist ontology—there is nothing missing.

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For the convergent potentialist to deny the actualist model is rather like accepting $\mathbb{R}$ but rejecting $\mathbb{C}$.

For this reason, I claim, there is little at stake in the dispute between convergent potentialism and actualism.

Convergent potentialism and actualism are two different views of the same subject matter.

A possible objection

Objection

The bi-interpretation is provable and sensible only from the actualist point of view.

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The potentialist can undertake the potentialist translation wholly within the potentialist ontology.

The potentialist thereby gains access to the full theory of what it is like in the actualist limit model.

Even the actualist agrees that this account is correct.

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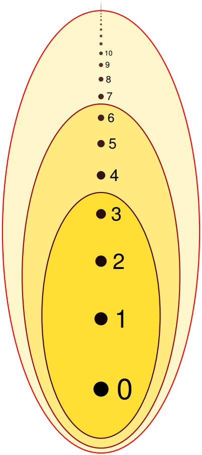
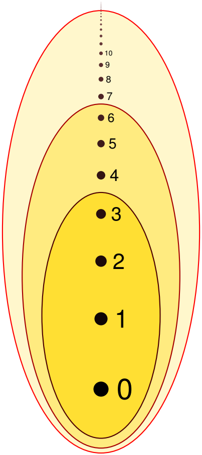
Situation is very similar with interpreting $\mathbb{C}$ in $\mathbb{R}$.

Implicit actualism in arithmetic potentialism



Consider classic Aristotelian potentialism.

Implicit actualism in arithmetic potentialism



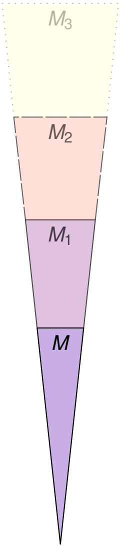
Consider classic Aristotelian potentialism.

This potentialist conception seems to arise from an actualist idea of what numbers are.

- The coherency of the potentialist conception presumes the coherency of the limit structure
- Built into the potentialist conception is an actualist conception of $\mathbb{N}$
- Actualist truth is interpretable via the potentialist translation.

This is the sense in which the view is implicitly actualist.

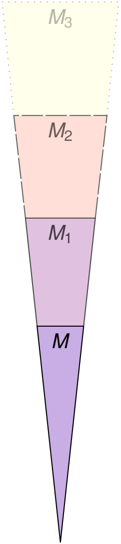
Rank potentialism, linear inevitability



Similarly with set-theoretic rank potentialism.

The rank potentialist can define truth in the limit model; and conversely.

Rank potentialism, linear inevitability



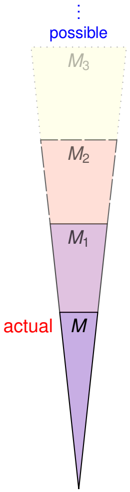
Similarly with set-theoretic rank potentialism.

The rank potentialist can define truth in the limit model; and conversely.

The two views—actualism versus potentialism—are two different, but equivalent perspectives on the same subject matter.

There seems little tension between this form of potentialism and actualism. No dispute between them.

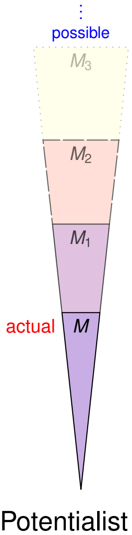
A misplaced analogy



Potentialist

Potentialist takes currently actual world as unfinished, continuing in further possible worlds.

A misplaced analogy



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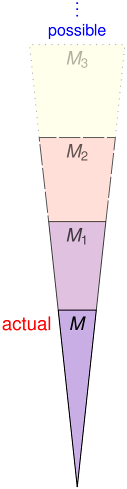

Potentialist

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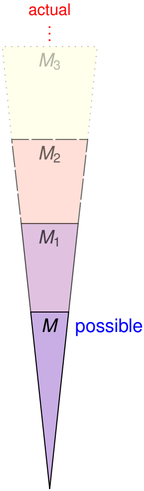
But actualist's actual world is not any world in system, but rather the limit of all of them.

Q: "Why aren't there more ordinals than the possible ordinals?"

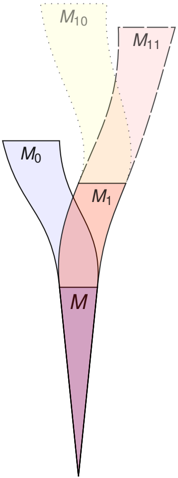
Potentialist and actualist seem in same boat.

Same issue for the universal set, Russell set.



Actualist

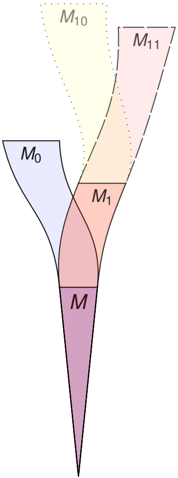
Divergent potentialism



The various divergent potentialist conceptions, in contrast, have a fundamentally different character.

In these systems, there is no actualist limit model to which the possible worlds are converging.

Divergent potentialism



The various divergent potentialist conceptions, in contrast, have a fundamentally different character.

In these systems, there is no actualist limit model to which the possible worlds are converging.

Here, the actualist world is a mirage, different every time the possible worlds are traversed.

In such a system, sitting at one of the possible worlds, the potentialist can seem to give no account of any final actual world.

This seems in many respects more genuinely potentialist.

Main philosophical conclusion

Convergent forms of potentialism can be seen in many respects as forms of actualism.

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They interpret actualist objects, structure, and truth.

Furthermore, the underlying conception of convergent potentialism seems to be grounded in a coherent conception of the limit model itself.

That picture is what I call implicitly actualist.

Divergent potentialism

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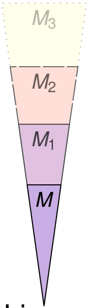
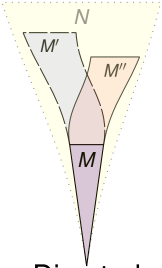
For these reasons, I find the central distinction to make in potentialism to be:

convergent potentialism versus divergent potentialism

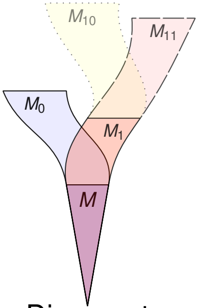
Varieties of potentialism



Linear inevitability
S4.3



Directed convergence
S4.2



Divergent potentialism
S4

Thank you.

Slides and articles available on http://jdh.hamkins.org.

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