What if your potentialism is implicitly actualist?

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University of Notre Dame

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Modal understanding of potentialism

The potentialist/actualist debate goes back to Aristotle
Modal understanding of potentialism

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Recent work emphasizes a modal understanding of potentialism

- System of possible worlds
- Currently actual universe fragment is one
- Modal operators $\Diamond \varphi, \Box \varphi$
Modal understanding of potentialism

The potentialist/actualist debate goes back to Aristotle

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- System of possible worlds
- Currently actual universe fragment is one
- Modal operators $\diamond \varphi$, $\Box \varphi$

Fundamental aspects of various potentialist conceptions are often revealed in the modally expressible features.

For any given potentialist conception, we should seek to find exactly the modal logic that is expressed.
Diverse potentialist conceptions

We are faced with an enormous range of distinct potentialist conceptions.

- Many different kinds of arithmetic potentialism
- Many different kinds of set-theoretic potentialism
Diverse potentialist conceptions

We are faced with an enormous range of distinct potentialist conceptions.

- Many different kinds of arithmetic potentialism
- Many different kinds of set-theoretic potentialism
- More general forms of potentialism, modal model theory, potentialist category theory

The various potentialist conceptions express subtly different philosophical ideas about the nature of potentialism and often have importantly different modal features.
My thesis

I shall argue several points.

- Many common forms of potentialism are implicitly actualist.
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- In these cases, the actualist and potentialist perspectives are bi-interpretable.
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- Consequently, for these forms of potentialism, there is little at stake in the actualism/potentialism dispute—they are simply two ways of looking at the same subject matter.
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- In these cases, the actualist and potentialist perspectives are bi-interpretable.
- Consequently, for these forms of potentialism, there is little at stake in the actualism/potentialism dispute—they are simply two ways of looking at the same subject matter.
- Meanwhile, other forms of potentialism are not like this.
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- Many common forms of potentialism are implicitly actualist.
- In these cases, the actualist and potentialist perspectives are bi-interpretable.
- Consequently, for these forms of potentialism, there is little at stake in the actualism/potentialism dispute—they are simply two ways of looking at the same subject matter.
- Meanwhile, other forms of potentialism are not like this.
- The central distinction in potentialism should therefore become: convergent potentialism vs. divergent.
Initial-segment potentialism in arithmetic

The classic potentialist conception

Aristotelian potentialism

Possible worlds consist of all numbers up to some $n$. 

$$u = \{0, 1, 2, \ldots, n\}$$
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Modal validities are exactly S4.3.
Nonlinear convergent arithmetic potentialism

Arbitrary set arithmetic potentialism

Possible worlds = any finite set of numbers
Large numbers may come into actuality before some smaller numbers.
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Kolmogorov potentialism

Numbers arrive in order of complexity of their descriptions. Kolmogorov complexity.
Googolplex $10^{10^{100}}$ comes early.
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Model-theoretic potentialism

Various model-theoretic potentialist conceptions arise by using models of $\mathsf{PA}$ as the possible worlds.

End-extensional potentialism, for example, expresses a potentialist conception of \textit{realms of feasibility}—speaks to ultrafinitism.
Various model-theoretic potentialist conceptions arise by using models of PA as the possible worlds.

End-extensional potentialism, for example, expresses a potentialist conception of *realms of feasibility*—speaks to ultrafinitism.

**Theorem (Hamkins [Ham18])**

The modal validities of arithmetic end-extensional potentialism are exactly S4.

The proof makes use of the universal algorithm.

What if your potentialism is implicitly actualist?

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Diverse forms of models-of-arithmetic potentialism

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Interpretability potentialism

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Submodel potentialism

Every world accesses the worlds of which it is a submodel. Validities are exactly S4.
## Diverse forms of models-of-arithmetic potentialism

### End-extensional potentialism

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And many more, [Ham18].
Enormous range of set-theoretic potentialism

Similarly we have numerous conceptions of set-theoretic potentialism.

Let me quickly mention several.
Set-theoretic rank potentialism

Possible worlds = $V_\alpha$, rank-initial segment of the universe

A set-theoretic analogue of Aristotelian potentialism.

Perhaps the canonical example of height potentialism

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Modal validities = S4.3.

S5 is valid at $V_\kappa$ for $\mathcal{L}_\in$ iff $\kappa$ is $\Sigma_3$-correct

S5 is valid at $V_\kappa$ for $\mathcal{L}_\Diamond$ iff $\kappa$ is fully correct.
Forcing potentialism

Worlds = models of set theory

Every world $M$ accesses its forcing extensions $M[G]$.

Canonical example of width potentialism.

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Forcing potentialism

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(Hamkins+Löwe [HL08]) The modal logic of forcing is exactly S4.2.
Forcing potentialism

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Canonical example of width potentialism.

(Hamkins+Löwe [HL08]) The modal logic of forcing is exactly S4.2.

(Hamkins [Ham03]) And yet, also consistent that a model validates S5 for sentences
Top-extensional potentialism

Possible worlds = models of set theory

World $M$ accesses its top extensions, $M \sqsubseteq N$.

A nonlinear form of height potentialism
Top-extensional potentialism

Possible worlds = models of set theory

World $M$ accesses its top extensions, $M \sqsubseteq N$.

A nonlinear form of height potentialism

(Hamkins+Woodin) Modal validities are exactly S4.

The proof uses a set-theoretic analogue of the universal algorithm: the universal $\Sigma_2$-definition
Set-theoretic end-extensional potentialism

Possible worlds = models of set theory

Each world accesses its end-extensions $M \subseteq N$, which means new sets appear, but no set gains new elements.

A hybrid height/width potentialism
Set-theoretic end-extensional potentialism

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Each world accesses its end-extensions $M \subseteq N$, which means new sets appear, but no set gains new elements.

A hybrid height/width potentialism

(Hamkins+Williams [HW21]) Modal validities are exactly S4.

There is divergent branching in this conception.

The proof uses a set-theoretic analogue of the universal algorithm: the universal $\Sigma_1$-definition.

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Submodel potentialism

Possible worlds = models of set theory

World $M$ accesses $N$ when $\langle M, \in^M \rangle$ is a submodel of $\langle N, \in^N \rangle$. 
Submodel potentialism

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In particular, sets can gain new elements! The empty set is not necessarily empty.
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(Hamkins [Ham13; HL22]) The modal validities are a surprise: exactly S4.3
Potentialist systems

A potentialist system is:

- A collection $\mathcal{W}$ of structures $M$ in a common language $\mathcal{L}$
- a reflexive transitive relation on these structures $\sqsubseteq$
- whenever $U \sqsubseteq W$, then $U$ is a substructure of $W$.

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Potentialist systems

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- whenever \( U \sqsubseteq W \), then \( U \) is a substructure of \( W \).

Alternative approach: \( U \sqsubseteq W \) means there is an embedding \( U \hookrightarrow W \). A system of counterparts.
A subtle point about accessibility

Do we want to say that $M$ accesses $N$ requires $M \subseteq N$?

Or should we focus on embeddings $M \hookrightarrow N$?
A subtle point about accessibility

Do we want to say that $M$ accesses $N$ requires $M \subseteq N$?

Or should we focus on embeddings $M \hookrightarrow N$?

Latter case leads to distinction between convergence & amalgamation

In my view, we would benefit from greater philosophical analysis of this distinction for potentialism.
### Direct extension vs. embedding access

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Models philosophical counterpart theory of individuals.
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Feature: algebraic systems often have robust amalgamation.

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### Direct extension vs. embedding accessibility

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#### Embedding accessibility

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Good news [HW24]: modal assertions in $\text{Mod}(T)$ are the same!

In fact [AD22]: the two Kripke structures are bisimilar.

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This limit structure remains outside the potentialist ontology.
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A potentialist system $\mathcal{W}$ is *convergent*, with limit $M$, if

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- Every world in $\mathcal{W}$ can be extended so as to accommodate any desired individual of $M$.

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The limit structure may have a fundamentally different nature than the possible worlds.
Linear inevitabilism

Convergent potentialism often arises because the possible worlds are linearly ordered, building up to a limit world in a linear coherent manner.
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\[ \Box \Diamond \varphi \rightarrow \Box \Box \Diamond \varphi \]

But also:

\[ \Diamond \varphi \land \Diamond \psi \rightarrow [\Diamond (\varphi \land \Diamond \psi) \lor \Diamond (\psi \land \Diamond \varphi)] \]

S4.3 is valid, and often exactly S4.3 is valid.
Nonlinear convergent potentialism

Other forms of potentialism are not linear, but nevertheless convergent.
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Still get validity of

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$$\lozenge \Box \varphi \rightarrow \Box \lozenge \varphi$$

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$$\lozenge \varphi \land \lozenge \psi \rightarrow [\lozenge (\varphi \land \lozenge \psi) \lor \lozenge (\psi \land \lozenge \varphi)]$$

S4.2 is valid, and often exactly S4.2.
Some forms of potentialism are not convergent.

Nonconvergence often occurs when a potentialist conception has divergent branching.

This is often a more radical form of potentialism.
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What is possible/necessary may depend on what’s already happened.

As objects become actual, they may close off some alternative possibilities.
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Validates S4, and often exactly only S4.
(Height v. width) versus (convergence v. divergence)
(Height v. width) versus (convergence v. divergence)

Some height potentialism is convergent

Aristotelian potentialism, rank potentialism. Linear and convergent.

What if your potentialism is implicitly actualist?

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Top-extensional potentialism in set theory. Also end-extensional arithmetic potentialism, viewed as finite set theory.
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## Introduction

- **Arithmetic potentialism**
- **Set theoretic potentialism**

## Potentialist systems

### Implicit actualism

- Joel David Hamkins

## Conclusion

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(Height v. width) versus (convergence v. divergence)

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Forcing potentialism, where generics are chosen as factors of a fixed limit model, collapsing everything (Steel, Scambler).

Some width potentialism is divergent

Forcing potentialism in the general case. Non-amalgamation phenomenon: extensions $M[c], M[d]$ with no common forcing extension. (And yet, S4.2 is valid.)

What if your potentialism is implicitly actualist?

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The potentialist translation

For any assertion \( \varphi \), define the *potentialist translation* \( \varphi^{\Diamond} \) by:
The potentialist translation

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- replace $\exists x$ with $\Diamond \exists x$
The potentialist translation

For any assertion $\varphi$, define the *potentialist translation* $\varphi$\$ by:

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**Theorem [HL22]**

With convergent potentialism, if a potentialist system $\mathcal{W}$ of possible worlds converges to limit model $M$, then

$$M \models \psi \quad \text{if and only if} \quad \mathcal{W} \models_{\mathcal{W}} \psi^\Diamond$$
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For any assertion \( \varphi \), define the *potentialist translation* \( \varphi\Diamond \) by:

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**Theorem [HL22]**

With convergent potentialism, if a potentialist system \( \mathcal{W} \) of possible worlds converges to limit model \( M \), then

\[
M \models \psi \quad \text{if and only if} \quad \mathcal{W} \models_{\mathcal{W}} \psi\Diamond
\]

Thus, actualist truth in the limit model is expressible within potentialism.

What if your potentialism is implicitly actualist?

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Convergent potentialism is implicitly actualist

Using only the potentialist ontology, therefore, the convergent potentialist has a full accounting of:

- actualist objects
- actualist structure
- actualist truth, via the potentialist translation
Convergent potentialism is implicitly actualist

Using only the potentialist ontology, therefore, the convergent potentialist has a full accounting of:

- actualist objects
- actualist structure
- actualist truth, via the potentialist translation

The potentialist denies the limit model exists, yet seems to know everything about it.

What if your potentialism is implicitly actualist?
Implicit actualism

The convergent potentialist can thus give a completely clear account of the actualist model—like interpreting $\mathbb{C}$ in $\mathbb{R}$.

The actualist model is interpretable in the potentialist ontology—there is nothing missing.
Implicit actualism

The convergent potentialist can thus give a completely clear account of the actualist model—like interpreting $C$ in $\mathbb{R}$.

The actualist model is interpretable in the potentialist ontology—there is nothing missing.

For the convergent potentialist to deny the actualist model is rather like accepting $\mathbb{R}$ but rejecting $C$.

For this reason, I claim, there is little at stake in the dispute between convergent potentialism and actualism.

Convergent potentialism and actualism are two different views of the same subject matter.
A possible objection

Objection

The bi-interpretation is provable and sensible only from the actualist point of view.
A possible objection

**Objection**

The bi-interpretation is provable and sensible only from the actualist point of view.

**Response**

The potentialist can undertake the potentialist translation wholly within the potentialist ontology.

The potentialist thereby gains access to the full theory of what it is like in the actualist limit model.

Even the actualist agrees that this account is correct.
A possible objection

**Objection**

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Even the actualist agrees that this account is correct.

Situation is very similar with interpreting $\mathbb{C}$ in $\mathbb{R}$. 

What if your potentialism is implicitly actualist?

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Implicit actualism in arithmetic potentialism

Consider classic Aristotelian potentialism.

What if your potentialism is implicitly actualist?

Joel David Hamkins
Implicit actualism in arithmetic potentialism

Consider classic Aristotelian potentialism.

This potentialist conception seems to arise from an actualist idea of what numbers are.

- The coherency of the potentialist conception presumes the coherency of the limit structure
- Built into the potentialist conception is an actualist conception of $\mathbb{N}$
- Actualist truth is interpretable via the potentialist translation.

This is the sense in which the view is implicitly actualist.
The rank potentialist can define truth in the limit model; and conversely.
Rank potentialism, linear inevitability

Similarly with set-theoretic rank potentialism.

The rank potentialist can define truth in the limit model; and conversely.

The two views—actualism versus potentialism—are two different, but equivalent perspectives on the same subject matter.

There seems little tension between this form of potentialism and actualism. No dispute between them.
A misplaced analogy

Potentialist takes currently actual world as unfinished, continuing in further possible worlds.

Potentialist

What if your potentialism is implicitly actualist?

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A misplaced analogy

Potentialist takes currently actual world as unfinished, continuing in further possible worlds.

This is how the potentialist answers the Q: “Why aren’t there more ordinals than the actual ordinals?”

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A misplaced analogy

Potentialist takes currently actual world as unfinished, continuing in further possible worlds.

This is how the potentialist answers the Q: “Why aren’t there more ordinals than the actual ordinals?”

But actualist’s actual world is not any world in system, but rather the limit of all of them.

Q: “Why aren’t there more ordinals than the possible ordinals?”

Potentialist and actualist seem in same boat.

Same issue for the universal set, Russell set.

What if your potentialism is implicitly actualist?

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Divergent potentialism

The various divergent potentialist conceptions, in contrast, have a fundamentally different character.

In these systems, there is no actualist limit model to which the possible worlds are converging.
Divergent potentialism

The various divergent potentialist conceptions, in contrast, have a fundamentally different character.

In these systems, there is no actualist limit model to which the possible worlds are converging.

Here, the actualist world is a mirage, different every time the possible worlds are traversed.

In such a system, sitting at one of the possible worlds, the potentialist can seem to give no account of any final actual world.

This seems in many respects more genuinely potentialist.
Main philosophical conclusion

Convergent forms of potentialism can be seen in many respects as forms of actualism.
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Convergent forms of potentialism can be seen in many respects as forms of actualism.

They interpret actualist objects, structure, and truth.
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Furthermore, the underlying conception of convergent potentialism seems to be grounded in a coherent conception of the limit model itself.
Main philosophical conclusion

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Furthermore, the underlying conception of convergent potentialism seems to be grounded in a coherent conception of the limit model itself.

That picture is what I call implicitly actualist.
Divergent potentialism

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Divergent potentialism

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For these reasons, I find the central distinction to make in potentialism to be:

convergent potentialism versus divergent potentialism
Varieties of potentialism

- Linear inevitability: S4.3
- Directed convergence: S4.2
- Divergent potentialism: S4

What if your potentialism is implicitly actualist?

Joel David Hamkins
Thank you.


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References I


References II


